The **luahyperbolic** latex package

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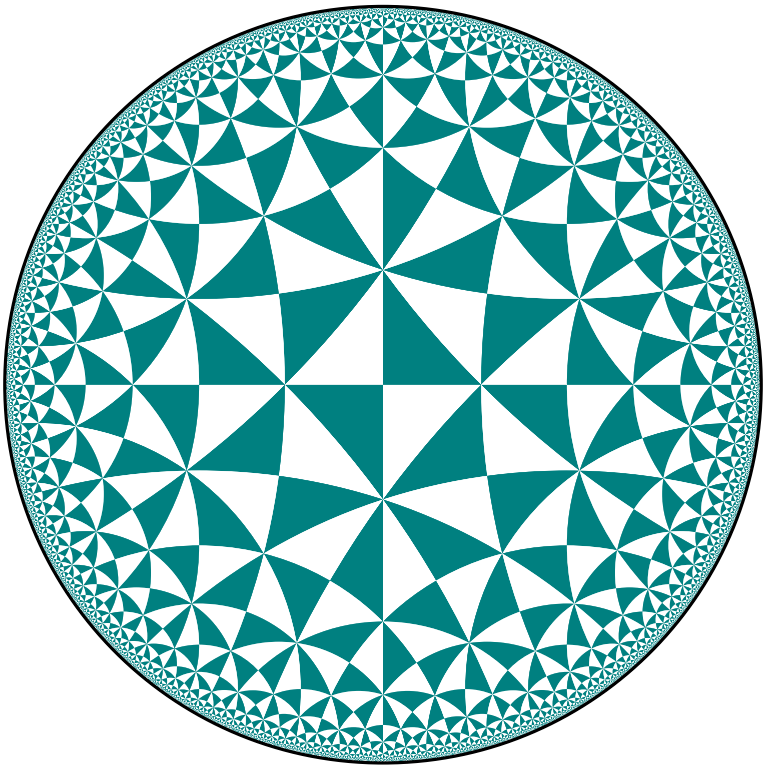


LaTeX code for above tiling : `\hyperbolicTiling{2}{4}{5}[scale=5,color=teal,depth=24]`
The latest version of this documentation is available at: github.com/dmegy/luahyperbolic.

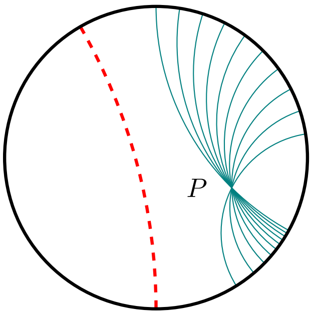
Contents

1 Introduction

1.1 What you can do with this package

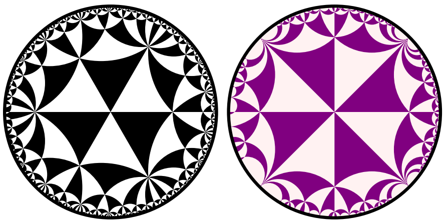
1. Draw hyperbolic geometry pictures, high precision (pdf output).
2. Use an easy syntax. At least, easier than with LaTeXmacros and bloody pgf.
3. Export generated pictures to external tikz files if you want.
4. Read the source code and learn hyperbolic geometry.
5. Compile **without** installing python packages or lua packages etc.
6. Compile **without** ”shell-escape” mode.
7. Compile everything in a single pass. No (Leeloo Dallas) multipass. Want to illustrate Euclid's fifth postulate ? Do it right there in your TeX file without having to include external files :

```
\begin{luacode*}
hyper.tikzBegin("scale=2")
local P = complex(0.5,-0.2)
local A = complex.exp_i(math.pi/20)
for k=1,10 do
  hyper.drawLine(P, A^k, "teal")
end
hyper.labelPoint(P, "$P$", "left=.2cm")
local style = "very thick, dashed, red"
hyper.drawLine(complex.J,-complex.I,style)
hyper.tikzEnd()
\end{luacode*}
```



8. Draw Tilings in one line, without any Lua code :

```
\hyperbolicTiling{3}{4}{7}[scale=1.4]
\hyperbolicTiling{4}{4}{inf}[scale=1.4,
    ↪ color=violet, backgroundcolor=pink
    ↪ !20, depth=6]
```



1.2 What this package cannot do for you

1. Really intensive computations, millions of geodesics etc. Source code is not optimized to keep maximum readability. All computations involve automorphisms. See other remarks below.
2. Coffee

1.3 Efficiency

For heavy computations, please use a ~~Python~~ C library. Nevertheless, this library is still reasonably efficient and usable:

- The first tiling illustration of this documentation (hyperbolic 2-4-5 tiling with 2361 geodesics) compiled in 4.72 seconds on a 2020 intel macbook air laptop.
- This entire documentation including all pictures compiles in approx. 20 seconds on the same machine, the majority of the time spent on the tilings. The package is suitable for producing lecture notes in hyperbolic geometry, which was the reason for all of this. For an entire book with dozens of pictures, you may need to externalize some of the pictures.
- Tests show that the bottleneck is TikZ, not Lua. In a future version, some future maintainer may add other fontends to the library, including drawing directly to the pdf with pdf primitives wich are several orders of magnitude faster than tikz.

1.4 How to load the package

Local install

Simply put the `luahyperbolic.sty` file in your working directory and add the following line to your tex preamble :

```
\usepackage{luahyperbolic}
```

The single file `luahyperbolic.sty` contains everything needed, including the necessary library for complex numbers. It can be sent to colleagues by email, etc.

It will declare two lua globals : `complex` and `hyper`, and define a `hyperbolicTiling` command for convenience.

If the sty file is not in the same directory as your tex file, you can write `\usepackage{../luahyperbolic}`.

1.5 Dependencies

The package luahyperbolic requires the following packages:

- tikz
- luacode

It loads them if they are not loaded already.

The latex files must be compiled with the **lualatex** engine.

1.6 Exporting the pictures

1.6.1 When finishing a picture

Simply add a filename to the closing `hyper.tikzEnd()` instruction, as below:

```
hyper.tikzEnd("myfile.tikz")
```

This will save the current picture (including `\begin{tikzpicture}[options]` and `\end{tikzpicture}`).

1.6.2 In the middle of the TikZpicture

It is also possible to export the current picture at any point with `hyper.tikzExport(optional-filename)`. If no filename is provided, the package will name it `hyper_picture_N` with `N` the picture counter. Luahyperbolic keeps count of how many pictures have been produced in the document. This allows exporting images in loops, or drawing several steps of the same picture.

1.7 (Advanced users) Build from source

The `luahyperbolic.sty` is pre-generated from a template sty file and four lua modules :

1. `complex.lua`
2. `luahyperbolic-core.lua`
3. `luahyperbolic-tikz.lua`
4. `luahyperbolic-tilings.lua`

These files are in the `lua/` subdirectory of the package repo.

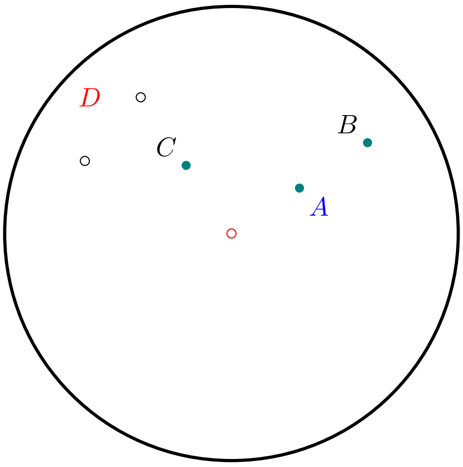
To build the `luahyperbolic.sty` from lua source files, execute `lua build.lua` in the root directory of the package. This will buid and replace the existing sty file.

2 Drawing and labeling elements

2.1 Draw and label points

2.1.1 Draw and label points

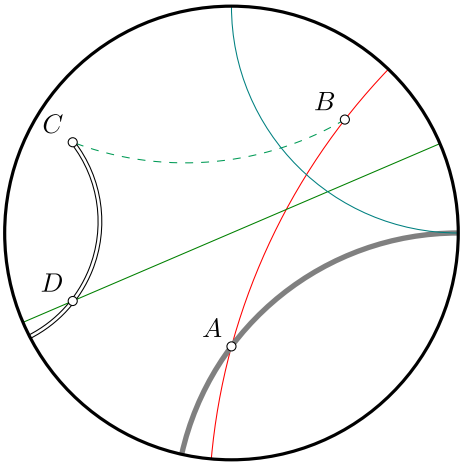
```
\begin{luacode*}
local A = complex(0.3,0.2)
local B = 2*A
local I, J = complex.I, complex.J
local C, D, E = I*A, I*B, J*B
hyper.tikzBegin()
hyper.drawPoint(0,"white, draw=red")
hyper.drawPoints(A, B, C, "teal")
hyper.drawPoints(D, E)
hyper.labelPoint(A,"$A$", "below right, blue")
hyper.labelPoints(B,"$B$", C, "$C$")
hyper.labelPoint(D,"$D$","left=.4cm, red")
hyper.tikzEnd()
\end{luacode*}
```



2.2 Draw/label/mark segments/lines/rays

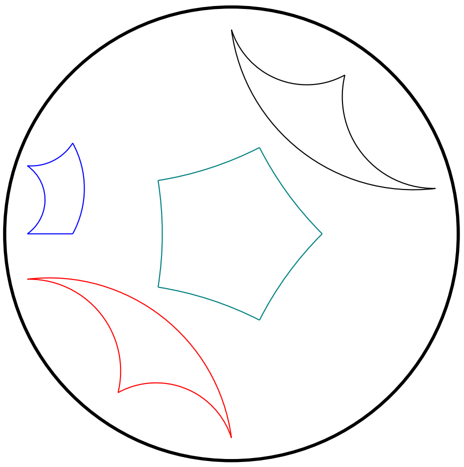
2.2.1 Segments lines and rays (through two points)

```
\begin{luacode*}
local A = -complex.I/2
local B = (1+complex.I)/2
local C = complex(-0.7,0.4)
local D = complex(-0.7,-0.3)
hyper.tikzBegin()
hyper.drawLine(A, B, "red")
hyper.drawSegment(B,C,"dashed, ForestGreen")
hyper.drawRay(C,D,"double,double distance=1pt"
    ↪ )
hyper.drawLine(D,-D, "green!50!black")
hyper.drawLine(A, 1, "gray, line width=2pt")
hyper.drawLine(1, complex.I, "teal")
hyper.drawPoints(A, B, C, D)
hyper.labelPoints(A, "$A$", B, "$B$", C, "$C$"
    ↪ , D, "$D$")
hyper.tikzEnd()
\end{luacode*}
```



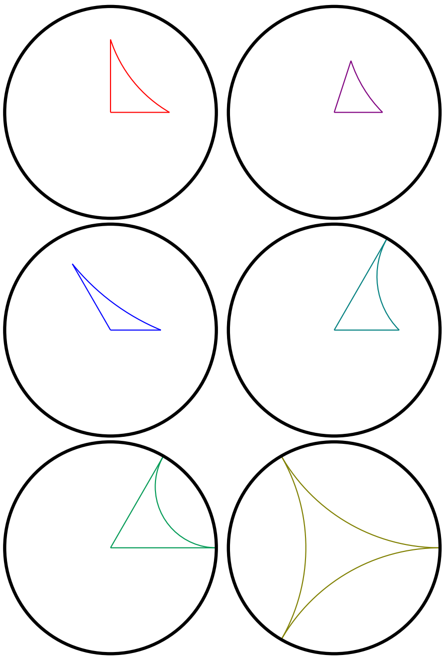
2.2.2 Triangles and polygons

```
\begin{luacode*}
local A = complex(.9,.2)
local B = complex(.5,.7)
local C = complex(0,.9)
hyper.tikzBegin()
hyper.drawTriangle(A, B, C)
hyper.drawTriangle(-A, -B, -C, "red")
hyper.drawPolygon(-.9, -.7, complex(-.7,.4),
    ↪ complex(-.9,.3), "blue")
local points={}
for k=1,5 do
  table.insert(points, complex.polar(.4,k*2*
      ↪ math.pi/5))
end
hyper.drawPolygonFromTable(points,"teal")
hyper.tikzEnd()
\end{luacode*}
```



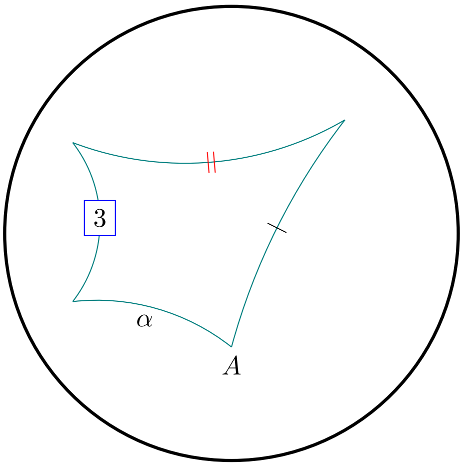
2.2.3 Triangles with prescribed angles

```
\begin{luacode*}
local function myTriangle(a, b, c, color)
  hyper.tikzBegin("scale=1.4")
  local A, B, C = hyper.triangleWithAngles(a,
      ↪ b, c)
  hyper.drawTriangle(A, B, C, color)
  hyper.tikzEnd()
end

local PI = math.pi

myTriangle(PI/2, PI/6, PI/10, "red")
myTriangle(2*PI/5, PI/4, PI/5, "red!50!blue")
tex.print("\\par")

myTriangle(2*PI/3, PI/8, PI/20, "blue")
myTriangle(PI/3, PI/4, 0, "blue!50!green")
tex.print("\\par")

myTriangle(PI/3, 0, 0, "ForestGreen")
myTriangle(0, 0, 0, "green!50!red")
\end{luacode*}
```



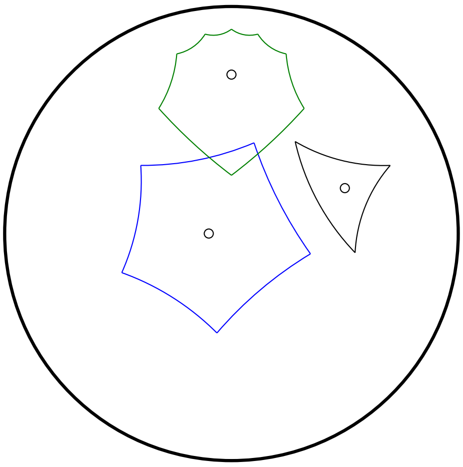
2.2.4 Label and mark segments

```
\begin{luacode*}
local A = -complex.I/2
local B = (1+complex.I)/2
local C = complex(-0.7,0.4)
local D = complex(-0.7,-0.3)
hyper.tikzBegin()
hyper.drawPolygon(A, B, C, D, "teal")
hyper.markSegment(A,B,"|")
local marking = "\\textcolor{red}{||}"
hyper.markSegment(B,C,marking)
hyper.labelSegment(C,D,"$3$", "fill=white,
    ↪ draw=blue")
hyper.labelSegment(D, A,"$\\alpha$", "below")
hyper.labelPoint(A, "$A$", "below")
hyper.tikzEnd()
\end{luacode*}
```



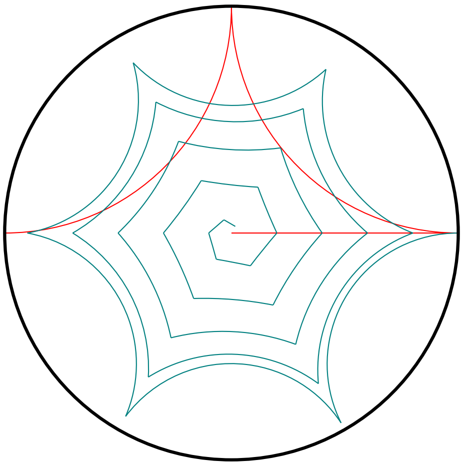
2.2.5 Regular polygons

```
\begin{luacode*}
local O1 = complex(.5,.2)
local A1 = complex(.7,.3)
local O2 = complex(-0.1,0)
local A2 = complex(-.4,.3)
local O3 = complex(0,.7)
local A3 = complex(0,.9)
hyper.tikzBegin()
hyper.drawRegularPolygon(O1, A1, 3)
hyper.drawRegularPolygon(O2, A2, 5, "blue")
hyper.drawRegularPolygon(O3, A3, 8, "green!50!
    ↪ black")
hyper.drawPoints(O1,O2,O3)
hyper.tikzEnd()
\end{luacode*}
```



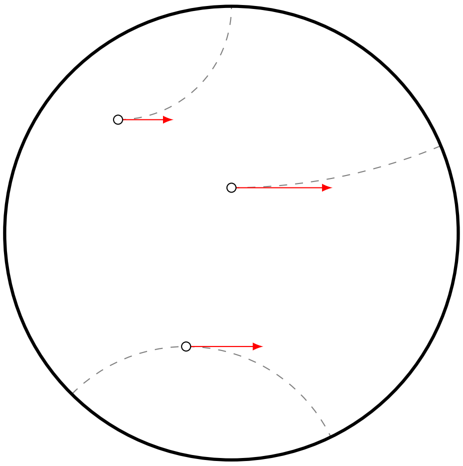
2.2.6 Polygonal lines

```
\begin{luacode*}
hyper.tikzBegin()
-- drawPolyline :
hyper.drawPolyline(0, 1, complex.I, -1, "red")
-- drawPolylineFromTable :
local points={}
local N=30
for k=1, N do
  table.insert(points, complex.polar(k/N, k*
      ↪ math.pi/3))
end
hyper.drawPolylineFromTable(points,"teal")
hyper.tikzEnd()
\end{luacode*}
```



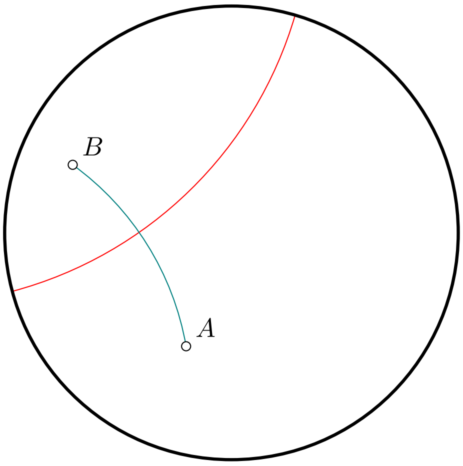
2.2.7 Lines and rays (point and tangent vector)

```
\begin{luacode*}
local A = complex(-0.2,-0.5)
local B = complex(-0.5,0.5)
local C = complex(0,0.2)
local v =  complex(1,0)
hyper.tikzBegin()
myVec = ">=latex, red"
myLine = "dashed, gray"
hyper.drawLineFromVector(A, v, myLine)
hyper.drawRayFromVector(B, v, myLine)
hyper.drawRayFromVector(C, v, myLine)
hyper.drawVector(A,v,myVec)
hyper.drawVector(B,v,myVec)
hyper.drawVector(C,v,myVec)
hyper.drawPoints(A, B,C)
hyper.tikzEnd()
\end{luacode*}
```



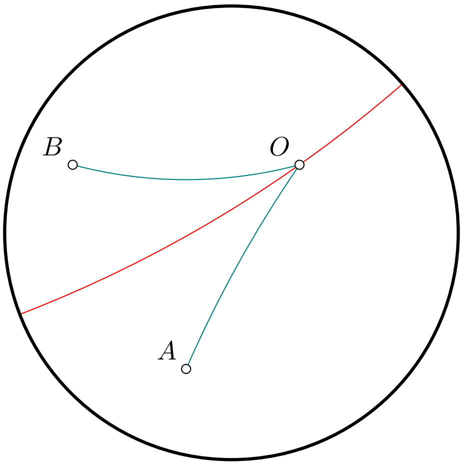
2.2.8 Perpendicular bisector

```
\begin{luacode*}
local A = complex(-0.2,-0.5)
local B = complex(-0.7,0.3)
hyper.tikzBegin()
hyper.drawPerpendicularBisector(A,B,"red")
hyper.drawSegment(A, B, "teal")
hyper.drawPoints(A,B)
hyper.labelPoints(A,"$A$", B, "$B$", "above
    ↪ right")
hyper.tikzEnd()
\end{luacode*}
```



2.2.9 Angle bisector

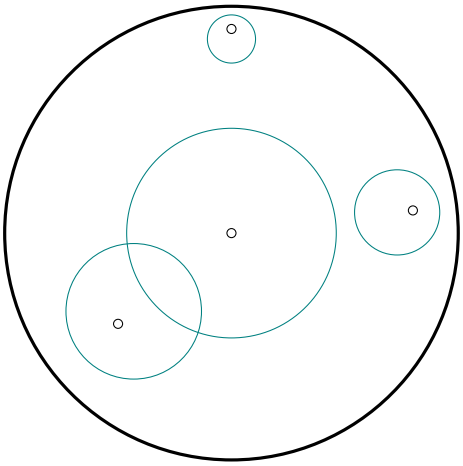
```
\begin{luacode*}
local A = complex(-0.2,-0.6)
local O = complex(0.3,0.3)
local B = complex(-0.7,0.3)
hyper.tikzBegin()
hyper.drawAngleBisector(A,O,B,"red")
hyper.drawPolyline(A,O,B, "teal")
hyper.drawPoints(A,O,B)
hyper.labelPoints(A, "$A$", O, "$O$", B, "$B$"
    ↪ )
hyper.tikzEnd()
\end{luacode*}
```



2.3 Circles, semicircles, arcs

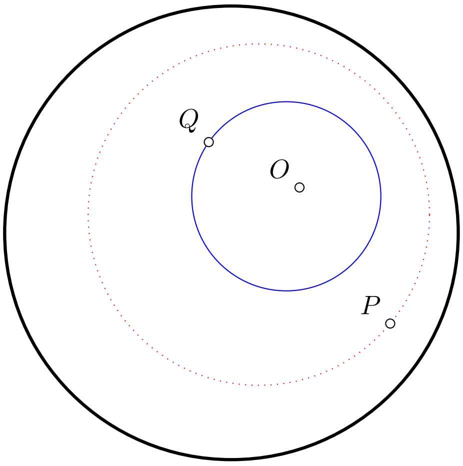
2.3.1 Center and radius

```
\begin{luacode*}
local points = {
  complex(0,0),
  complex(-0.5,-0.4),
  complex(0.8,0.1),
  complex(0,0.9)
}
hyper.tikzBegin()
for _,point in ipairs(points) do
  hyper.drawCircle(point,1,"teal")
  hyper.drawPoint(point)
end
-- NOTE : all circles have same radius=1 !
hyper.tikzEnd()
\end{luacode*}
```



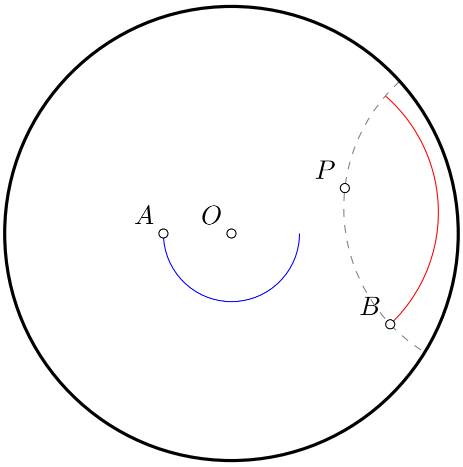
2.3.2 Circle through point

```
\begin{luacode*}
local O = complex(0.3,0.2)
local P = complex(0.7,-0.4)
local Q = complex(-0.1,0.4)
hyper.tikzBegin()
hyper.drawCircleThrough(O, P, "dotted, red")
-- or draw by hand :
hyper.drawCircle(O, hyper.distance(O,Q), "blue
    ↪ ")
hyper.drawPoints(O, P, Q)
hyper.labelPoints(O, "$O$", P, "$P$", Q, "$Q$"
    ↪ )
hyper.tikzEnd()
\end{luacode*}
```



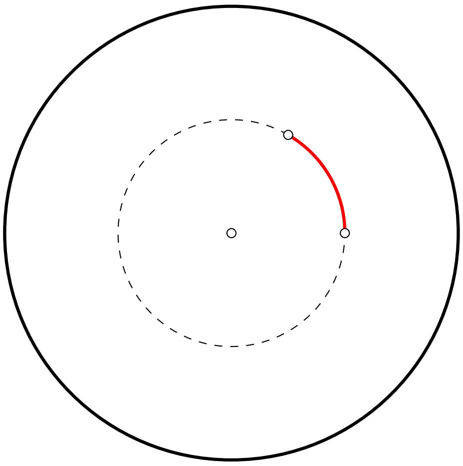
2.3.3 Semicircles

```
\begin{luacode*}
local O = complex(0,0)
local A = complex(-0.3,0)
local P = complex(0.5,0.2)
local B = complex(0.7,-0.4)
hyper.tikzBegin()
hyper.drawSemicircle(O, A, "blue")
hyper.drawSemicircle(P, B, "red")
hyper.labelPoints(O, "$O$", A, "$A$", P, "$P$"
    ↪ , B, "$B$")
hyper.drawLine(P,B, "gray, dashed")
hyper.drawPoints(O, A, P, B)
hyper.tikzEnd()
\end{luacode*}
```



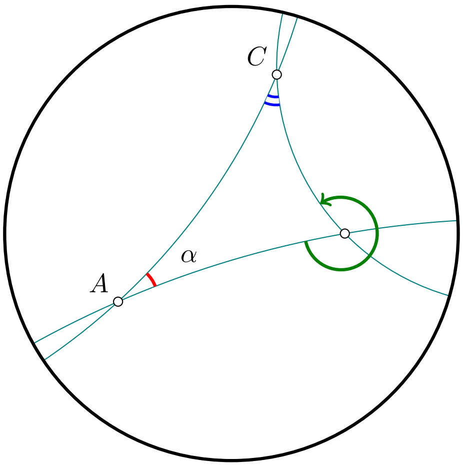
2.3.4 Arc (center and endpoints)

```
\begin{luacode*}
local O = complex(0,0)
local A = complex(0.5,0)
local B = complex.polar(0.5,math.pi/3)
hyper.tikzBegin()
hyper.drawArc(O, A, B, "very thick, red")
hyper.drawArc(O, B, A, "dashed")
hyper.drawPoints(O,A,B)
hyper.tikzEnd()
\end{luacode*}
```



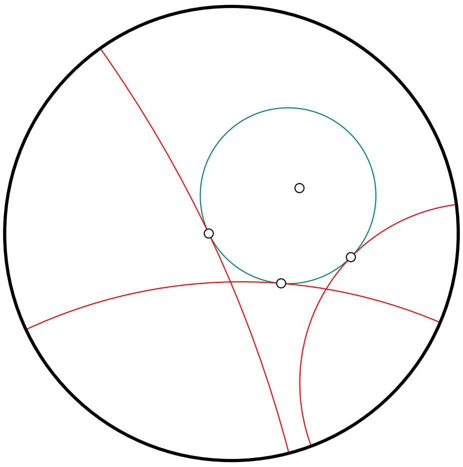
2.3.5 Angles

```
\begin{luacode*}
local A = complex(-.5,-.3)
local B = complex(0.5,0)
local C = complex(0.2,.7)
hyper.tikzBegin()
hyper.drawLines(A, B, B, C, C, A, "teal")
hyper.drawAngle(B, A, C, "red, very thick")
hyper.drawAngle(A, C, B, "thick, double,
    ↪ double distance=2pt, blue")
hyper.drawAngle(A, B, C, "very thick, ->,
    ↪ green!50!black")
hyper.drawPoints(A, B, C)
hyper.labelPoints(A, "$A$", C, "$C$")
hyper.labelPoint(N, "$B$", "above right")
-- poor man's labelAngle:
hyper.labelPoint(A,"$\\alpha$", "above=.6cm,
    ↪ right=.7cm")
hyper.tikzEnd()
\end{luacode*}
```



2.3.6 Tangent (line) at point

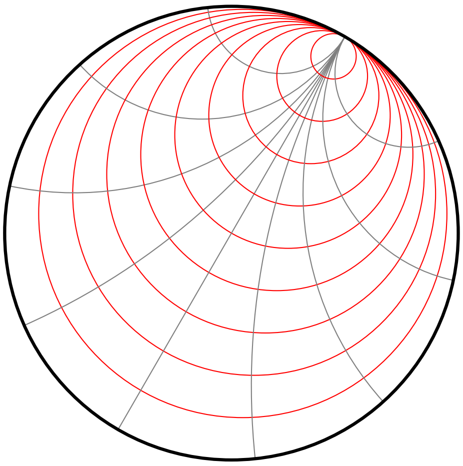
```
\begin{luacode*}
local O = complex(0.3,0.2)
local P = complex(-0.1,0)
local phi  =  hyper.rotation(O,math.pi/4)
local Q =phi(P)
local R = phi(Q)
hyper.tikzBegin()
hyper.drawCircleThrough(O, P, "teal")
hyper.drawTangentAt(O,P,"red")
hyper.drawTangentAt(O,Q,"red")
hyper.drawTangentAt(O,R,"red")
hyper.drawPoints(O, P, Q, R)
hyper.tikzEnd()
\end{luacode*}
```



2.4 Horocycles and hypercycles

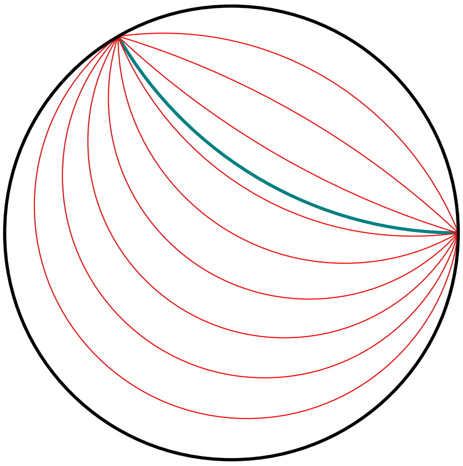
2.4.1 Horocycles

```
\begin{luacode*}
local theta=math.pi/3
local idealPoint = complex.exp_i(theta)
local N = 10
hyper.tikzBegin()
for k=1,N-1 do
  local point = idealPoint * (2*k/N-1)
  hyper.drawHorocycle(idealPoint,point,"red")
  -- and an orthogonal geodesic :
  local endPoint = complex.exp_i(theta+2*k*
      ↪ math.pi/N)
  hyper.drawLine(idealPoint,endPoint,"gray")
end
hyper.tikzEnd()
\end{luacode*}
```



2.4.2 Hypercycles

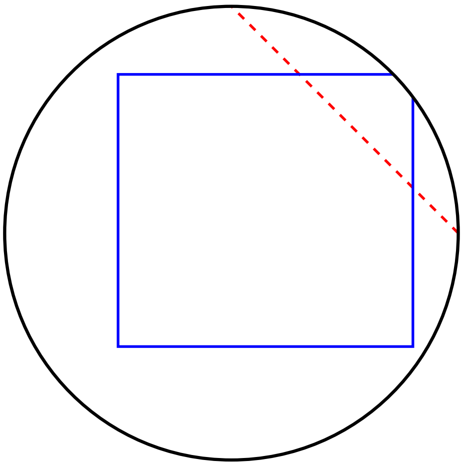
```
\begin{luacode*}
hyper.tikzBegin()
local A = 1
local B = complex.J
hyper.drawLine(A,B,"very thick, teal")
local N = 10
for k=1,N-1 do
  local P = (2*k/N-1)*(-B^2)
  hyper.drawHypercycleThrough(P, A, B, "red")
end
hyper.tikzEnd()
\end{luacode*}
```



2.5 Printing custom strings to TikZ

In case you want to insert custom TikZ instructions into the TikZ picture, the library provides a `tikzPintf()` function, which is essentially `tex.print` composed with `string.format()`, but also saves the printed string to the tikz buffer.

```
\begin{luacode*}
local A = complex.ONE
local B = complex.I
local C = (-A-B)/2
hyper.tikzBegin()
hyper.tikzPrintf(
  "\\draw[%s] (%f,%f) -- (%f,%f) ;",
  "red, thick, dashed", A.re, A.im, B.re, B.im
)
hyper.tikzPrintf(
  "\\draw[%s] (%f,%f) rectangle (%f,%f) ;",
  "blue, thick", C.re, C.im, 0.8,0.7
)
hyper.tikzEnd()
\end{luacode*}
```



The package doesn't expose euclidean drawing functions. Users should write TikZ directly in the picture as above with `tikzPrintf`, or use some other library, for example the amazing library `tkz-euclide`.

2.6 Modifying default styles

The drawing default styles are defined in lua constants that the user can modify. Keys and default values include:

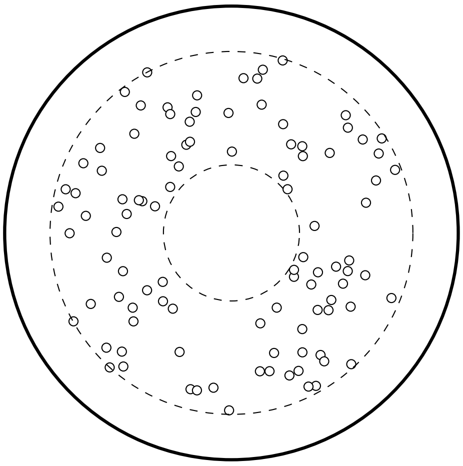
```
hyper.LABEL_STYLE = "above␣left"
hyper.DRAW_POINT_RADIUS = 0.02
hyper.DRAW_POINT_STYLE = "white,␣draw=black"
hyper.BOUNDARY_CIRCLE_STYLE = "very␣thick,␣black"
```

For a complete list, see the source file `luahyperbolic-tikz.lua`

3 Defining points

3.1 Random points with uniform density

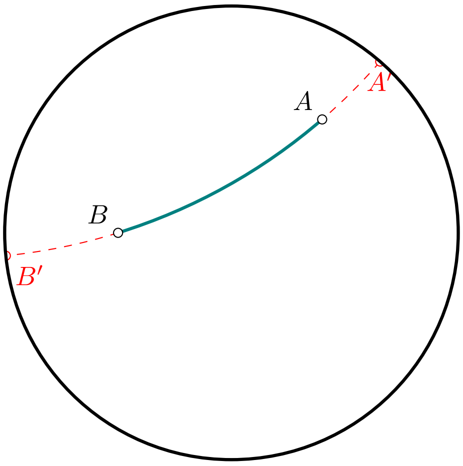
```
\begin{luacode*}
local rMin, rMax = .3, .8
hyper.tikzBegin()
for k=1,100 do
  local M = hyper.randomPoint(rMin,rMax)
  hyper.drawPoint(M)
end
hyper.tikzPrintf("\\draw[dashed] (0,0) circle
    ↪ (%s) ;", rMin)
hyper.tikzPrintf("\\draw[dashed] (0,0) circle
    ↪ (%s) ;", rMax)
hyper.tikzEnd()
\end{luacode*}
```



3.2 Endpoints, midpoint, interpolate

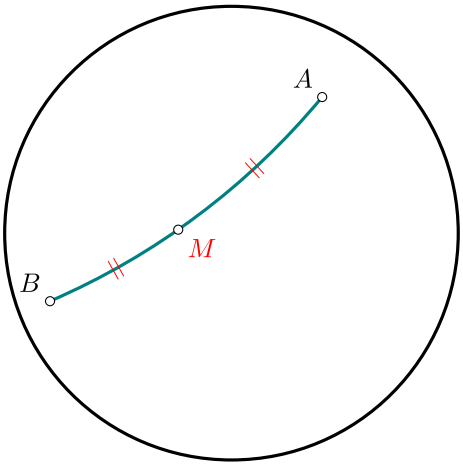
3.2.1 Endpoints

```
\begin{luacode*}
local A = complex(.4,.5)
local B = complex(-.5,0)
local AA, BB = hyper.endpoints(A,B)
hyper.tikzBegin()
hyper.drawSegment(A,B, "very thick, teal")
hyper.drawSegments(AA,A, B, BB, "dashed, red")
hyper.labelPoints(A, "$A$", B, "$B$")
hyper.labelPoint(AA, "$A'$", "red, below")
hyper.labelPoint(BB, "$B'$", "red, below right
    ↪ ")
hyper.drawPoints(A, B)
hyper.drawPoints(AA,BB,"white, draw=red")
hyper.tikzEnd()
\end{luacode*}
```



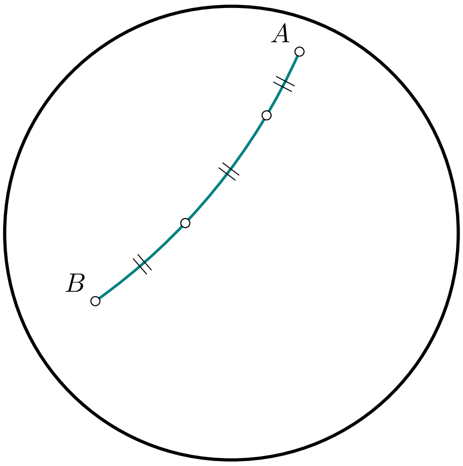
3.2.2 Midpoint

```
\begin{luacode*}
local A = complex(0.4,0.6)
local B = complex(-0.8,-0.3)
local M = hyper.midpoint(A,B)
hyper.tikzBegin()
hyper.drawSegment(A,B, "very thick, teal")
hyper.drawPoints(A, B, M)
hyper.labelPoints(A, "$A$", B, "$B$")
hyper.labelPoint(M, "$M$", "below right, red")
local marking = "\\textcolor{red}{||}"
hyper.markSegment(A,M,marking)
hyper.markSegment(M,B,marking)
hyper.tikzEnd()
\end{luacode*}
```



3.2.3 Interpolate and barycenter

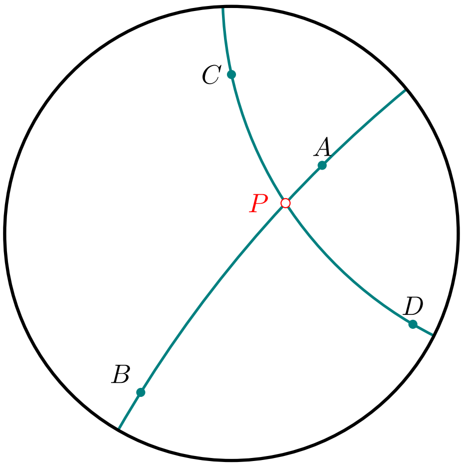
```
\begin{luacode*}
local A = complex(0.3,0.8)
local B = complex(-0.6,-0.3)
local P1 = hyper.interpolate(A, B, 1/3)
local P2 = hyper.interpolate(A, B, 2/3)
hyper.tikzBegin()
hyper.drawSegment(A,B, "thick, teal")
hyper.labelPoints(A, "$A$", B, "$B$")
hyper.markSegments(A, P1, P1, P2, P2, B, "||")
hyper.drawPoints(A, B, P1, P2)
hyper.tikzEnd()
\end{luacode*}
```



3.3 Intersections

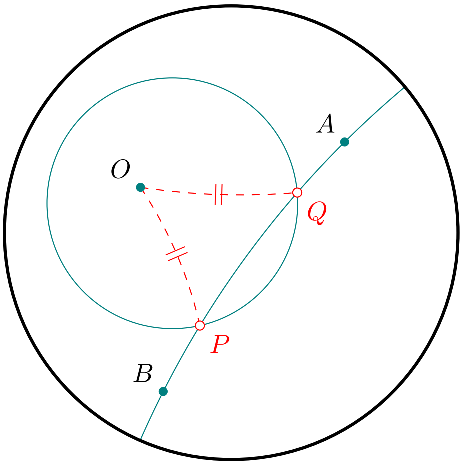
3.3.1 Line-Line (LL) intersection

```
\begin{luacode*}
local A = complex(0.4,0.3)
local B = complex(-0.4,-0.7)
local C = complex(0,0.7)
local D = complex(0.8,-0.4)
local P = hyper.interLL(A, B, C, D)
hyper.tikzBegin()
hyper.drawLines(A, B, C, D, "thick, teal")
hyper.drawPoints(A, B, C, D,"thick, teal")
hyper.drawPoint(P,"white, draw=red")
hyper.labelPoint(A, "$A$", "above")
hyper.labelPoint(B, "$B$")
hyper.labelPoint(C, "$C$", "left")
hyper.labelPoint(D, "$D$", "above")
hyper.labelPoint(P, "$P$", "left=.1cm, red")
hyper.tikzEnd()
\end{luacode*}
```



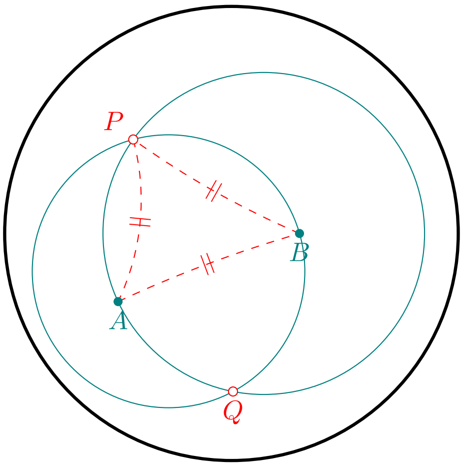
3.3.2 Line-circle (LC) intersection

```
\begin{luacode*}
local A = complex(0.5,0.4)
local B = complex(-0.3,-0.7)
local O = complex(-0.4,0.2)
local radius = 1.5
local P, Q = hyper.interLC(A, B, O, radius)
hyper.tikzBegin()
hyper.drawLine(A, B, "teal")
hyper.drawCircle(O,radius, "teal")
hyper.drawSegments(O, P, O, Q, "red, dashed")
hyper.drawPoints(A, B, O, "teal")
hyper.drawPoints(P, Q, "white, draw=red")
hyper.labelPoints(A, "$A$", B, "$B$", O, "$O$"
    ↪ )
hyper.labelPoints(P, "$P$", Q, "$Q$", "red,
    ↪ below right")
local mark = "\\textcolor{red}{||}"
hyper.markSegments(O, P, O, Q, mark)
hyper.tikzEnd()
\end{luacode*}
```



3.3.3 Circle-circle (CC) intersection

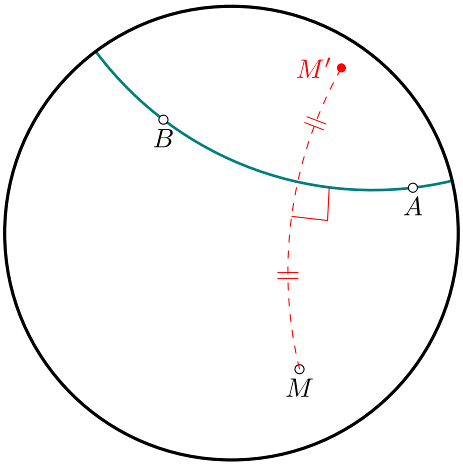
```
\begin{luacode*}
local A = complex(-0.5,-.3)
local B = complex(0.3,0)
local r = hyper.distance(A, B)
local P, Q = hyper.interCC(A, r, B, r)
hyper.tikzBegin()
hyper.drawCircle(A, r, "teal")
hyper.drawCircle(B, r, "teal")
hyper.drawPolygon(A, B, P, "dashed, red")
hyper.drawPoints(P, Q, "white, draw=red")
hyper.drawPoints(A, B, "teal")
hyper.labelPoints(A, "$A$", B, "$B$", "below,
    ↪ teal")
hyper.labelPoints(P, "$P$", "above left, red")
hyper.labelPoint(Q, "$Q$", "red, below")
local mark = "\\textcolor{red}{||}"
hyper.markSegments(A, B, B, P, P, A, mark)
hyper.tikzEnd()
\end{luacode*}
```



3.4 Reflections and projections

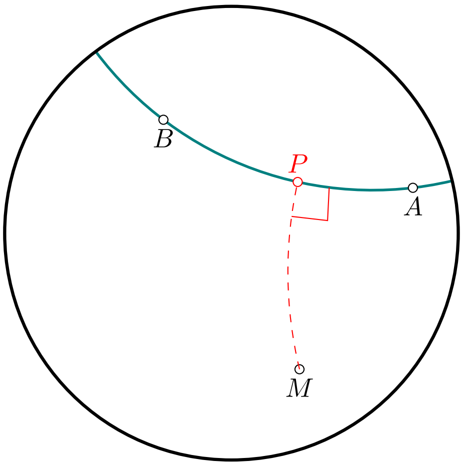
3.4.1 Reflection across a geodesic line

```
\begin{luacode*}
local A = complex(0.8,0.2)
local B = complex(-0.3,0.5)
local M = complex(0.3,-0.6)
local Mp = (hyper.reflection(A,B))(M)
hyper.tikzBegin()
hyper.drawLine(A, B, "thick, teal")
hyper.drawPoints(A, B, M)
hyper.drawPoint(Mp,"red")
hyper.drawSegment(M,Mp,"dashed,red")
hyper.labelPoint(Mp,"$M'$", "left, red")
local P = hyper.interLL(A, B, M, Mp)
local mark="\\textcolor{red}{||}"
hyper.labelPoints(A, "$A$", B, "$B$", M, "$M$"
    ↪ , "below")
hyper.markSegments(M, P, P, Mp, mark)
hyper.drawRightAngle(M, P, A, "red")
hyper.tikzEnd()
\end{luacode*}
```



3.4.2 Projection onto a geodesic line

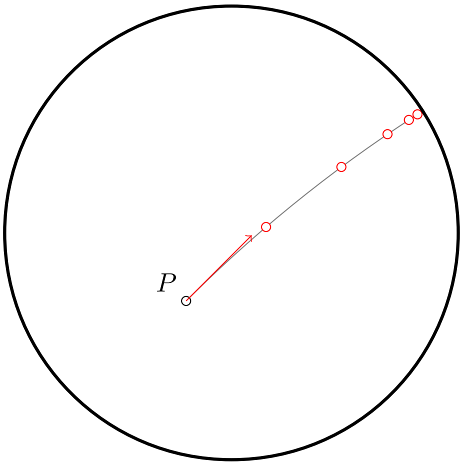
```
\begin{luacode*}
local A = complex(0.8,0.2)
local B = complex(-0.3,0.5)
local M = complex(0.3,-0.6)
local P = (hyper.projection(A,B))(M)
hyper.tikzBegin()
hyper.drawLine(A, B, "thick, teal")
hyper.drawPoints(A, B, M)
hyper.drawPoint(P,"white, draw=red")
hyper.drawSegment(M,P,"dashed,red")
hyper.labelPoint(P,"$P$", "above, red")
hyper.labelPoints(A, "$A$", B, "$B$", M, "$M$"
    ↪ , "below")
hyper.drawRightAngle(M,P,A,"red")
hyper.tikzEnd()
\end{luacode*}
```



3.5 Exponential map

3.5.1 Image of a vector by exponentiel map at a point

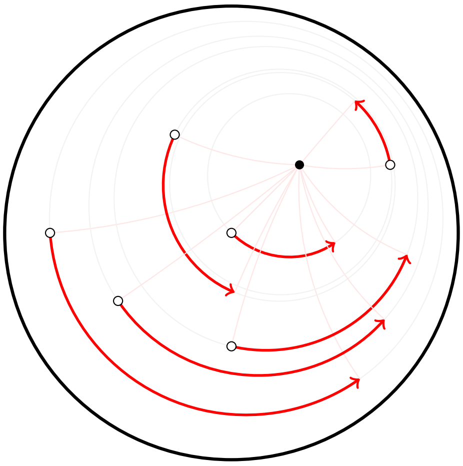
```
\begin{luacode*}
local P = complex(-.2,-0.3)
local vect = complex.exp_i(math.pi/4)
hyper.tikzBegin()
hyper.drawRayFromVector(P, vect, "gray")
for k=1,5 do
  local A = hyper.expMap(P,k*vect)
  hyper.drawPoint(A, "white, draw=red")
end
hyper.drawPoint(P)
hyper.labelPoint(P,"$P$")
hyper.drawVector(P,vect, "red")
hyper.tikzEnd()
\end{luacode*}
```



3.6 Automorphisms (direct isometries)

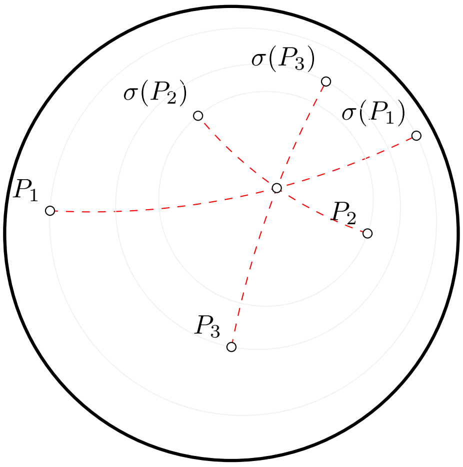
3.6.1 (Elliptic) Rotation by 60°

```
\begin{luacode*}
local O = complex(0.3,0.3)
local r = hyper.rotation(O,math.pi/3)
local points = {0, -.8, complex.J/2, -complex.
    ↪ I/2, complex(-.5,-.3), complex(.7,.3)}
hyper.tikzBegin()
for _,P in ipairs(points) do
  local rP = r(P)
  hyper.drawCircleThrough(O,P, "gray!10")
  hyper.drawArc(O,P,rP,"->, red, thick")
  hyper.drawPolyline(P,O,rP, "red!10")
  hyper.drawPoint(P)
end
hyper.drawPoint(O,"black")
hyper.tikzEnd()
\end{luacode*}
```



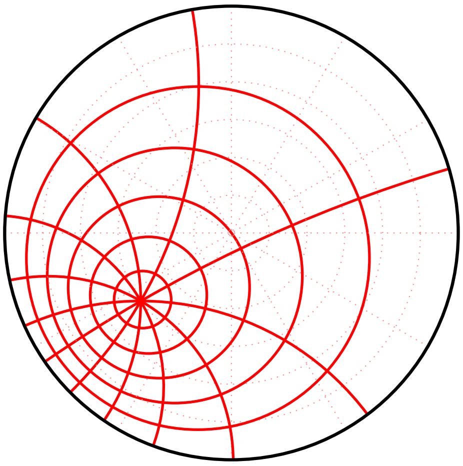
3.6.2 Symmetry around a point (rotation by $180°$)

```
\begin{luacode*}
local O = complex(0.2,0.2)
local s = hyper.symmetry(O)
local points={complex(-.8,.1), complex(.6,0),
    ↪ complex(0,-.5)}
hyper.tikzBegin()
for k, P in ipairs(points) do
  hyper.drawCircleThrough(O, P, "gray!10")
  local sP = s(P)
  hyper.drawSegment(P,sP, "red, dashed")
  hyper.drawPoints(P,sP)
  hyper.labelPoint(P, "$P_{".. k .."}$")
  hyper.labelPoint(sP, "$\\sigma(P_{".. k .."
      ↪ })$")
end
hyper.drawPoint(O)
hyper.tikzEnd()
\end{luacode*}
```



3.6.3 Hyperbolic automorphims

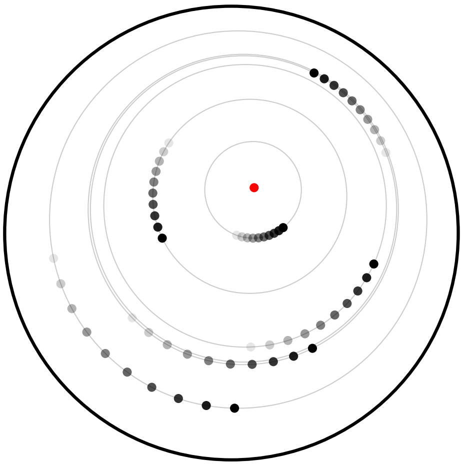
```
\begin{luacode*}
local A = complex(0.4,0.3)
local phi = hyper.automorphism(A,0)
local N=6
hyper.tikzBegin()
style1 = "dotted, red!50"
style2 =  "thick, red"
for k=1,N do
  local P = complex.polar(k/N, k*math.pi/N)
  local Q = phi(P)
  hyper.drawLine(0,P,style1)
  hyper.drawLine(-A,Q,style2)
  if k < N then
    hyper.drawCircleThrough(0,P, style1)
    hyper.drawCircleThrough(-A,Q,style2)
  end
end
hyper.tikzEnd()
\end{luacode*}
```



3.7 Orbits under automorphisms

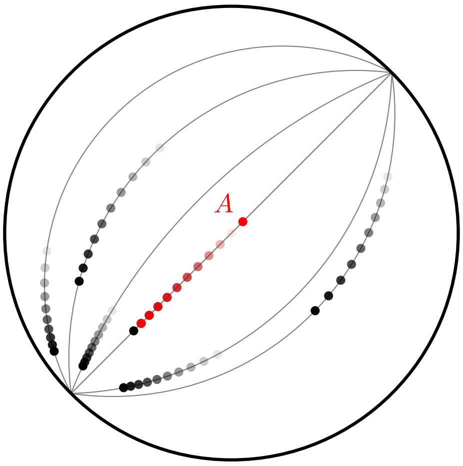
3.7.1 Orbit under elliptic automorphism (rotation)

```
\begin{luacode*}
local nbIterations = 10
local O = complex(0.1,0.2)
local points = {0, -.8, complex.J/2, -complex.
    ↪ I/2, complex(-.5,-.3), complex(.7,.3)}
local rot = hyper.rotation(O,math.pi/30) --
    ↪ angle in radians
hyper.tikzBegin()
hyper.drawPoint(O,"red")
for _,P in ipairs(points) do
  hyper.drawCircleThrough(O,P,"draw opacity
      ↪ =0.2")
  hyper.drawPointOrbit(P, rot, nbIterations)
end
hyper.tikzEnd()
\end{luacode*}
```



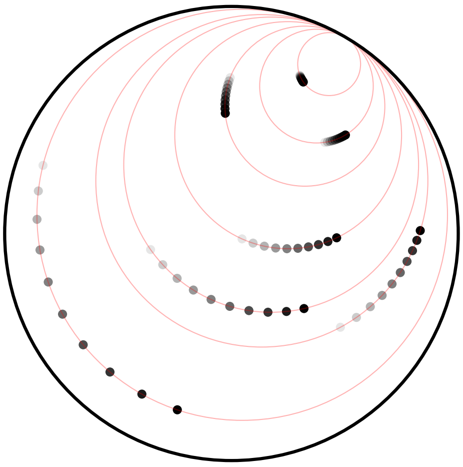
3.7.2 Orbit under hyperbolic automorphism

```
\begin{luacode*}
local points = {0, -.8, complex.J/2, -complex.
    ↪ I/2, complex(-.5,-.3), complex(.7,.3)}
local nbIterations = 10
local A = complex(0.05,0.05)
local phi = hyper.automorphism(A,0)
local X, Y = hyper.endpoints(A,0)
hyper.tikzBegin()
hyper.drawLine(A,0,"red!50")
for _,P in ipairs(points) do
  hyper.drawHypercycleThrough(P,X,Y,"gray")
  hyper.drawPointOrbit(P, phi, nbIterations)
end
hyper.drawPoint(A,"red")
hyper.labelPoint(A,"$A$", "above left, red")
hyper.drawPointOrbit(A, phi, nbIterations, "
    ↪ red")
hyper.tikzEnd()
\end{luacode*}
```



3.7.3 Orbit under parabolic automorphism

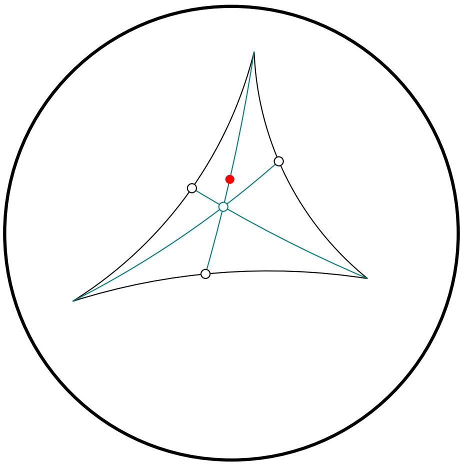
```
\begin{luacode*}
local nbIterations = 10
local limit = complex.exp_i(math.pi/3) -- on
    ↪ circle
local phi = hyper.parabolic(limit,math.pi/30)
local points={
  complex(0,0), complex(0,.7),
  complex(.4,.4), complex(.3,.7),
  complex(-.8,.4), complex(-.4,0),
  complex(.4,-.45)}
hyper.tikzBegin()
for _,P in ipairs(points) do
  hyper.drawPointOrbit(P, phi, nbIterations)
  hyper.drawHorocycle(limit, P, "red, draw
      ↪ opacity=.3")
end
hyper.tikzEnd()
\end{luacode*}
```



3.8 Triangle geometry (CAUTION)

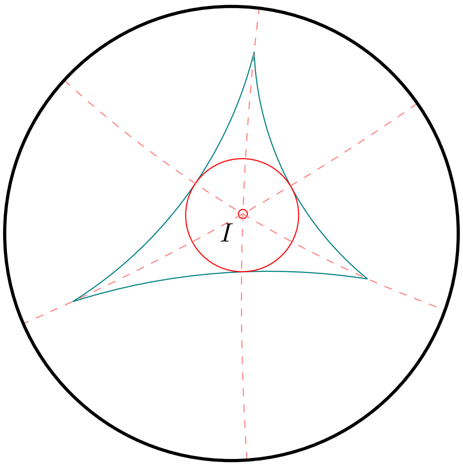
3.8.1 Centroid (intersection of medians)

```
\begin{luacode*}
hyper.tikzBegin()
local A = complex(0.1,0.8)
local B = complex(-0.7,-0.3)
local C = complex(0.6,-0.2)
local AA = hyper.midpoint(B,C)
local BB = hyper.midpoint(C,A)
local CC = hyper.midpoint(A,B)
local G = hyper.triangleCentroid(A,B,C)
hyper.drawTriangle(A,B,C)
hyper.drawSegments(A, AA, B, BB, C, CC, "teal"
    ↪ )
hyper.drawPoints(AA,BB,CC)
hyper.drawPoint(G,"white, draw=teal")
-- CAUTION :
local wrong = hyper.interpolate(A,AA,2/3)
hyper.drawPoint(wrong, "red")
hyper.tikzEnd()
\end{luacode*}
```



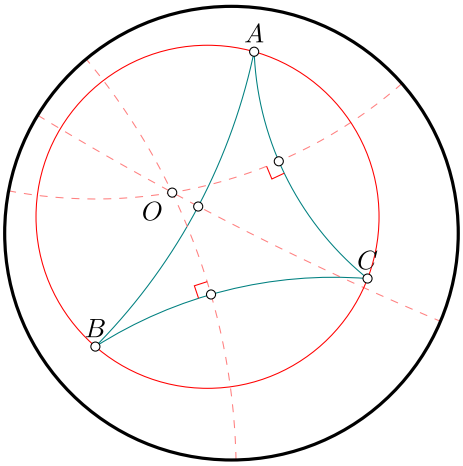
3.8.2 Incenter and incircle

```
\begin{luacode*}
hyper.tikzBegin()
local A = complex(0.1,0.8)
local B = complex(-0.7,-0.3)
local C = complex(0.6,-0.2)
local I = hyper.triangleIncenter(A,B,C)
hyper.drawTriangle(A,B,C,"teal")
hyper.drawPoint(I,"white, draw=red")
hyper.labelPoint(I,"$I$", "below left")
hyper.drawIncircle(A,B,C,"red")
-- show intersection :
local mystyle = "dashed, red!50"
hyper.drawAngleBisector(A,B, C, mystyle)
hyper.drawAngleBisector(B,C,A, mystyle)
hyper.drawAngleBisector(C,A,B, mystyle)
hyper.tikzEnd()
\end{luacode*}
```



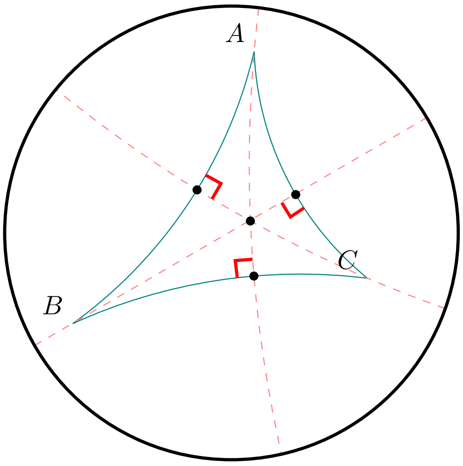
3.8.3 Circumcenter and circumcircle (CAUTION : does not always exist)

```
\begin{luacode*}
local A = complex(0.1,0.8)
local B = complex(-0.6,-0.5)
local C = complex(0.6,-0.2)
local O = hyper.triangleCircumcenter(A,B,C)
local BC = hyper.midpoint(B,C)
local CA = hyper.midpoint(C,A)
local AB = hyper.midpoint(A,B)
hyper.tikzBegin()
local mystyle = "dashed, red!50"
hyper.drawTriangle(A,B,C,"teal")
hyper.drawPerpendicularBisector(A, B, mystyle)
hyper.drawPerpendicularBisector(B, C, mystyle)
hyper.drawPerpendicularBisector(C, A, mystyle)
hyper.drawCircumcircle(A,B,C,"red")
hyper.labelPoint(O,"$O$", "below left")
hyper.drawRightAngle(O, CA, C,"red",1/8)
hyper.drawRightAngle(O,BC,B,"red",1/8)
hyper.drawPoints(O,A,B,C,AB,BC,CA)
hyper.labelPoints(A, "$A$", B, "$B$", C, "$C$"
    ↪ ,"above")
hyper.tikzEnd()
\end{luacode*}
```



3.8.4 Orthocenter (CAUTION : does not always exist)

```
\begin{luacode*}
local A = complex(0.1,0.8)
local B = complex(-0.7,-0.4)
local C = complex(0.6,-0.2)
local H = hyper.triangleOrthocenter(A,B,C)
local AA = hyper.projection(B,C)(A)
local BB = hyper.projection(C,A)(B)
local CC = hyper.projection(A,B)(C)

hyper.tikzBegin()
local mystyle = "dashed, red!50"
local style2 = "very thick, red"
hyper.drawTriangle(A,B,C,"teal")
hyper.drawPerpendicularThrough(A,B,C,mystyle)
hyper.drawPerpendicularThrough(B,C,A,mystyle)
hyper.drawPerpendicularThrough(C,A,B,mystyle)
hyper.drawRightAngle(A,AA,B,style2,1/15)
hyper.drawRightAngle(B,BB,C,style2,1/8)
hyper.drawRightAngle(C,CC,A,style2,1/12)
hyper.labelPoints(A, "$A$", B, "$B$", C, "$C$"
    ↪ )
hyper.drawPoints(AA,BB,CC,H,"above")
hyper.tikzEnd()
\end{luacode*}
```

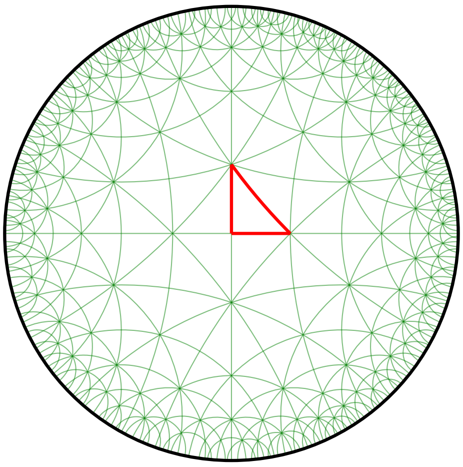


4 Tilings

4.1 Draw Tilings

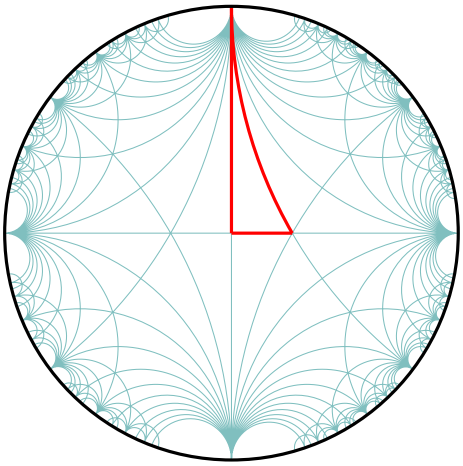
4.1.1 Triangle tiling (almost) by hand

```
\begin{luacode*}
local PI = math.pi
local A, B, C = hyper.triangleWithAngles(PI/2,
    ↪  PI/4, PI/5)
local geodesics = {
  {hyper.endpoints(A,B)},
  {hyper.endpoints(B,C)},
  {hyper.endpoints(C,A)}
}
geodesics = hyper.propagateGeodesics(geodesics
    ↪ , 12) -- depth = 12
hyper.tikzBegin()
hyper.drawLinesFromTable(geodesics, "green!50!
    ↪ black, draw opacity=.5")
hyper.drawTriangle(A,B,C,"red, very thick")
hyper.tikzEnd()
\end{luacode*}
```



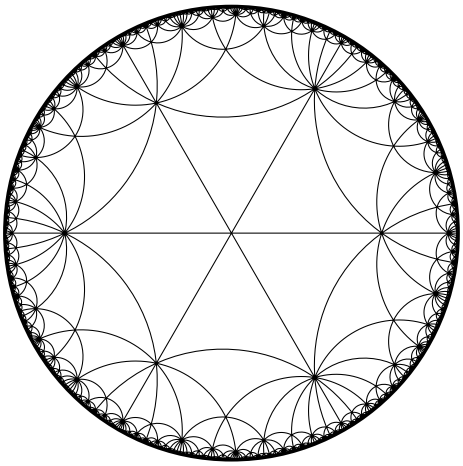
4.1.2 Triangle tiling with less code

```
\begin{luacode*}
local PI = math.pi
local DEPTH = 12
local A, B, C = hyper.triangleWithAngles(PI/2,
    ↪ PI/3,0)
hyper.tikzBegin()
hyper.drawTilingFromTriangle(A, B, C, DEPTH, "
    ↪ teal, draw opacity=.5")
hyper.drawTriangle(A,B,C,"red, very thick")
hyper.tikzEnd()
\end{luacode*}
```



4.1.3 Triangle tiling with even less code

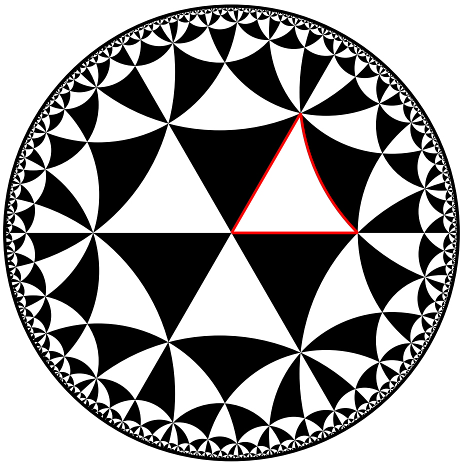
```
\begin{luacode*}
local PI = math.pi
hyper.tikzBegin()
hyper.drawTilingFromAngles(PI/3,PI/5,PI/7,12)
hyper.tikzEnd()
\end{luacode*}
```



4.2 Fill tilings

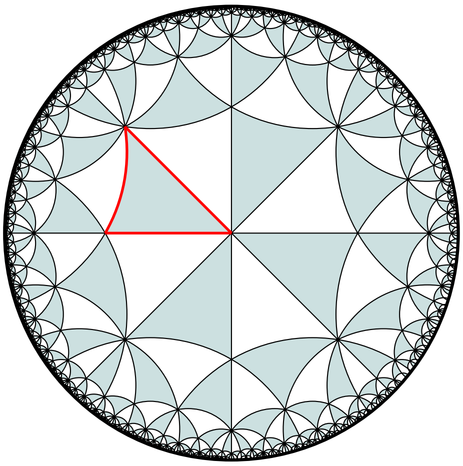
4.2.1 The 3–4–5 triangle tiling

```
\begin{luacode*}
local PI = math.pi
local DEPTH = 10
local A, B, C = hyper.triangleWithAngles(PI/3,
    ↪  PI/4, PI/5)
hyper.tikzBegin()
hyper.fillTilingFromTriangle(A, B, C, DEPTH)
-- default color is black
hyper.drawPolygon(A, B, C, "thick, red")
hyper.tikzEnd()
\end{luacode*}
```



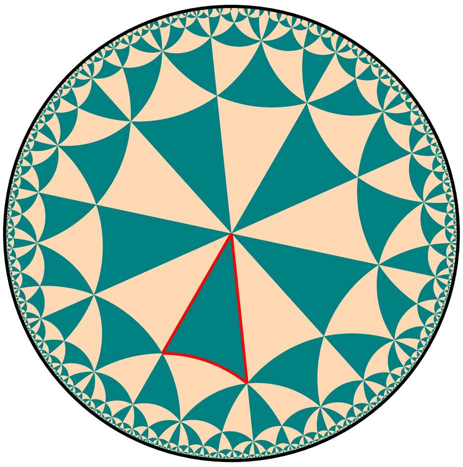
4.2.2 Same 3–4–5 tiling centered on point B

```
\begin{luacode*}
local PI = math.pi
local DEPTH = 10
local A, B, C = hyper.triangleWithAngles(PI/3,
    ↪  PI/4, PI/5)
local phi = hyper.automorphism(B)
A, B, C = phi(A), phi(B), phi(C) -- put B in
    ↪ the center
hyper.tikzBegin()
hyper.fillTilingFromTriangle(A, B, C, DEPTH, "
    ↪ fill opacity=.2, teal!80!black, draw=
    ↪ black")
hyper.drawPolygon(A, B, C, "thick, red")
hyper.tikzEnd()
\end{luacode*}
```



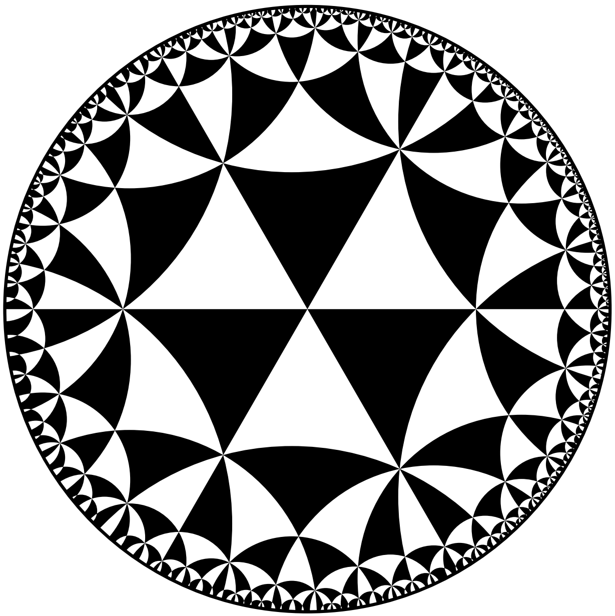
4.2.3 Same 3–4–5 tiling centered on point C

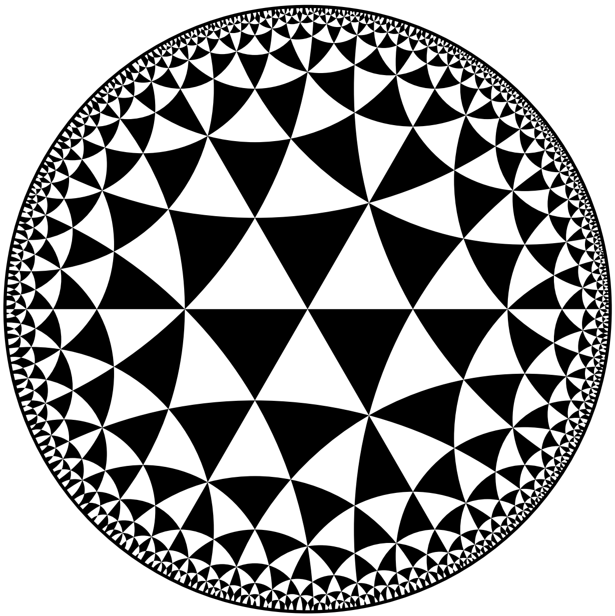
```
\begin{luacode*}
local PI = math.pi
local DEPTH = 10
local A, B, C = hyper.triangleWithAngles(PI/3,
    ↪  PI/4, PI/5)
local phi = hyper.automorphism(C)
A, B, C = phi(A), phi(B), phi(C) -- put C in
    ↪ the center
hyper.tikzBegin()
hyper.tikzPrintf("\\fill[orange!30] (0,0)
    ↪ circle (1);")
hyper.fillTilingFromTriangle(A, B, C, DEPTH, "
    ↪ teal")
hyper.drawPolygon(A, B, C, "thick, red")
hyper.tikzEnd()
\end{luacode*}
```

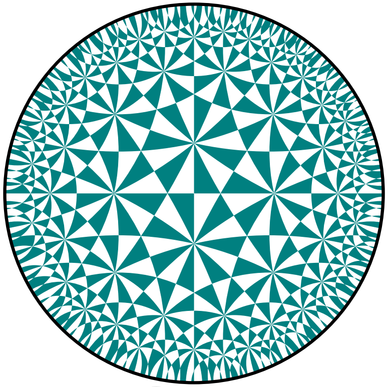


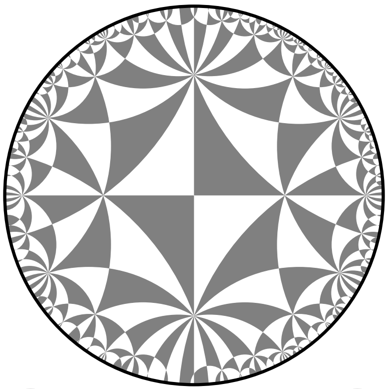
4.3 Tilings without any Lua code

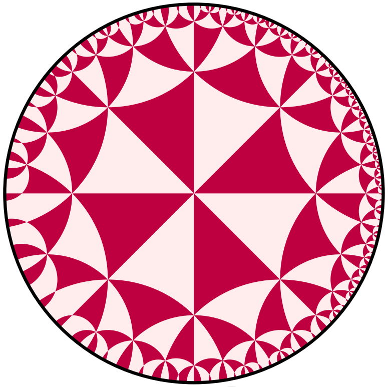
The package provides a TeX macro for users who want to completely avoid Lua.

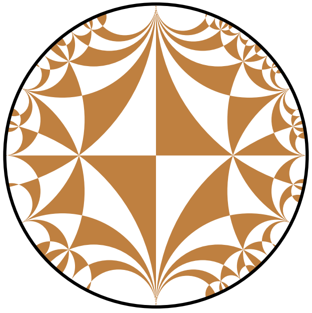


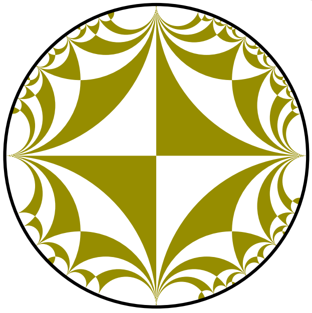


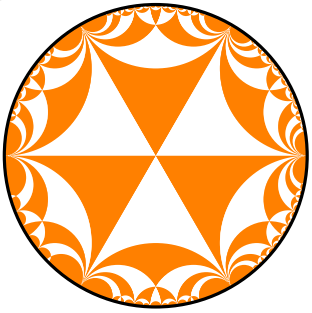


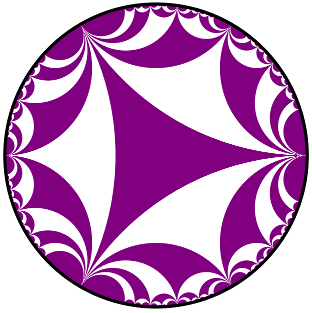












The syntax is `\hyperbolicTiling{p}{q}{r}[options]` with p, q, r positive integers or `inf` satisfying $\frac{1}{p} + \frac{1}{q} + \frac{1}{r} < 1$. The associated hyperbolic triangle has angles $\frac{\pi}{p}$, $\frac{\pi}{q}$, and $\frac{\pi}{r}$.

The above tilings were obtained with the following commands:

```
\hyperbolicTiling{3}{4}{5}[scale=4]
\hyperbolicTiling{3}{3}{4}[scale=4,depth=12]
\hyperbolicTiling{2}{3}{7}[scale=2.5, color=teal, depth=16]
\hyperbolicTiling{2}{5}{9}[scale=2.5, color=gray]
\hyperbolicTiling{4}{4}{4}[scale=2.5, depth=6, color=purple,backgroundcolor=pink!30]
\hyperbolicTiling{2}{5}{inf}[scale=2, depth=6, color=brown]
\hyperbolicTiling{2}{inf}{inf}[scale=2, depth=6, color=olive]
\hyperbolicTiling{3}{inf}{inf}[scale=2, depth=6, color=orange]
\hyperbolicTiling{inf}{inf}{inf}[scale=2, depth=5, color=violet]
```

Default values are : scale=3, depth=8, color=black, backgroundcolor=white.

5 Overview of the lua libraries

The Lua files are in the `lua/` subdirectory of the repo.

To build the `luahyperbolic.sty` from the lua source files, run `lua build.lua` on the `lua/` directory. (Lua must be installed.)

The Lua source files are not needed to compile a latex document using luahyperbolic, only the sty file is necessary.

5.1 complex.lua

Functions

__add
__div
__eq
__mul
__pow
__sub
__tostring
__unm
abs
abs2
arg
clone
coerce
conj
det
distinct
dot
exp
exp_i
invert
isClose
isColinear
isComplex
isImag
isInteger
isNear
isNearInteger
isNot
isReal
isUnit
log
new
nonzero
polar
rotate
rotate90
scal
toPolar
unit

Constants (numbers and complex numbers)

EPS, EPS_INV, I, J, ONE, ZERO, __index,

5.2 luahyperbolic

The luahyperbolic library consists of three submodules : luahyperbolic-core, luahyperbolic-tikz and luahyperpolic-tilings. These three submodules are flattened in a single `hyper` global module. (The submodules are still available in three tables hyper.core, hyper.tikz and hyper.tilings if necessary.)

Functions beginning with underscore are available in the module but should not be needed for normal use. Don't use them, they could become private in the future and not be available to users anymore.

Functions

_assert
_assert_in_closed_disk
_assert_in_disk
_circle_to_euclidean
_coerce_assert_in_closed_disk
_coerce_assert_in_disk
_ensure_order
_error
_geodesic_data
_in_closed_disk
_in_disk
_in_half_plane
_metric_factor
_midpoint_to_origin
_on_circle
_on_line
_on_segment
_radial_half
_radial_scale
_warning
angle
automorphism
automorphismFromPairs
automorphismSending
barycenter2
distance
distanceToLine
drawAngle
drawAngleBisector
drawArc
drawCircle
drawCircleRadius
drawCircleThrough
drawCircumcircle
drawHorocycle
drawHypercycleThrough
drawIncircle
drawLine
drawLineFromVector
drawLines
drawLinesFromTable
drawPerpendicularBisector
drawPerpendicularThrough
drawPoint
drawPointOrbit
drawPoints
drawPolygon
drawPolygonFromTable
drawPolyline
drawPolylineFromTable
drawRay
drawRayFromPoints
drawRayFromVector
drawRegularPolygon
drawRightAngle
drawSegment
drawSegments
drawSemicircle
drawTangentAt
drawTilingFromAngles
drawTilingFromTriangle
drawTriangle
drawVector
endpoints
endpointsAngleBisector
endpointsPerpendicularBisector
expMap
fillTilingFromAngles
fillTilingFromTriangle
interCC
interLC
interLL
interpolate
labelPoint
labelPoints
labelSegment
markSegment
markSegments
midpoint
mobiusTransformation
parabolic

pointOrbit
projection
propagateGeodesics
randomPoint
reflection
rotation
rotationFromPair
symmetry
symmetryAroundMidpoint
tangentVector
tikzBegin
tikzClearBuffer
tikzDefineNodes
tikzEnd
tikzExport
tikzGetFirstLines
tikzPrintf
tikz_shape_closed_line
tikz_shape_euclidean_segment
tikz_shape_segment
triangleCentroid
triangleCircumcenter
triangleIncenter
triangleOrthocenter
triangleWithAngles

Constants

Numbers : DRAW_ANGLE_DIST, DRAW_POINT_RADIUS, EPS, QUANTIZATION_SCALE, tikzNbPicturesExported,

Strings : ANGLE_STYLE, BOUNDARY_CIRCLE_STYLE, CIRCLE_STYLE, DRAW_POINT_STYLE, GEODESIC_STYLE, HOROCYCLE_STYLE, HYPERCYCLE_STYLE, LABEL_STYLE, MARKING_SIZE, MARKING_STYLE, TIKZ_BEGIN_DISK, module, tikzpictureOptions,

Tables : core, tikz, tikzBuffer, tilings,

6 Known bugs

1. Cannot use the TeX macro `\hyperbolicTiling` inside a `begin{center}` environment. Boxes and `\centering` don't work well either.
2. This documentation sometimes errors during compilation with error `pgf : dimension too large` but a second compilation always succeeds.

7 Index

Index of functions used in this documentation